Progress on fluid computations of Hall thrusters: first-principles model for anomalous transport and dynamics effects

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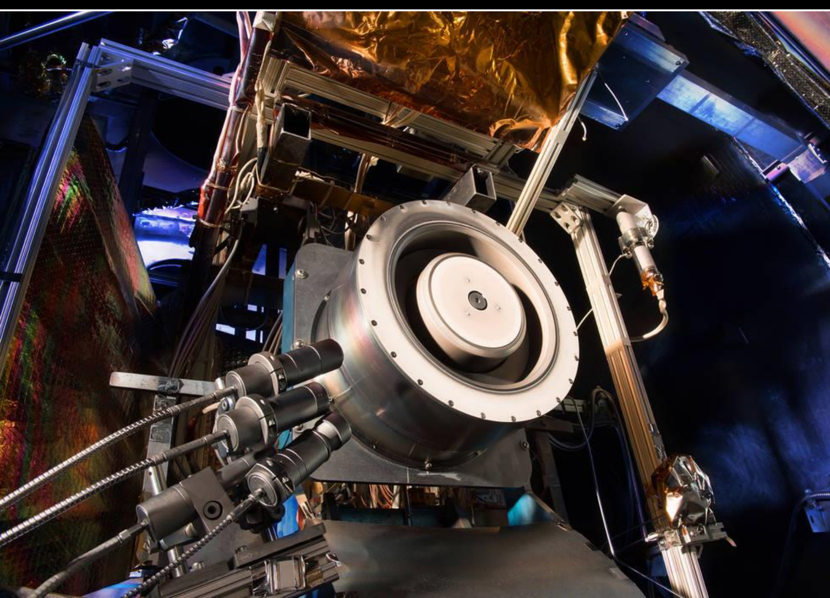
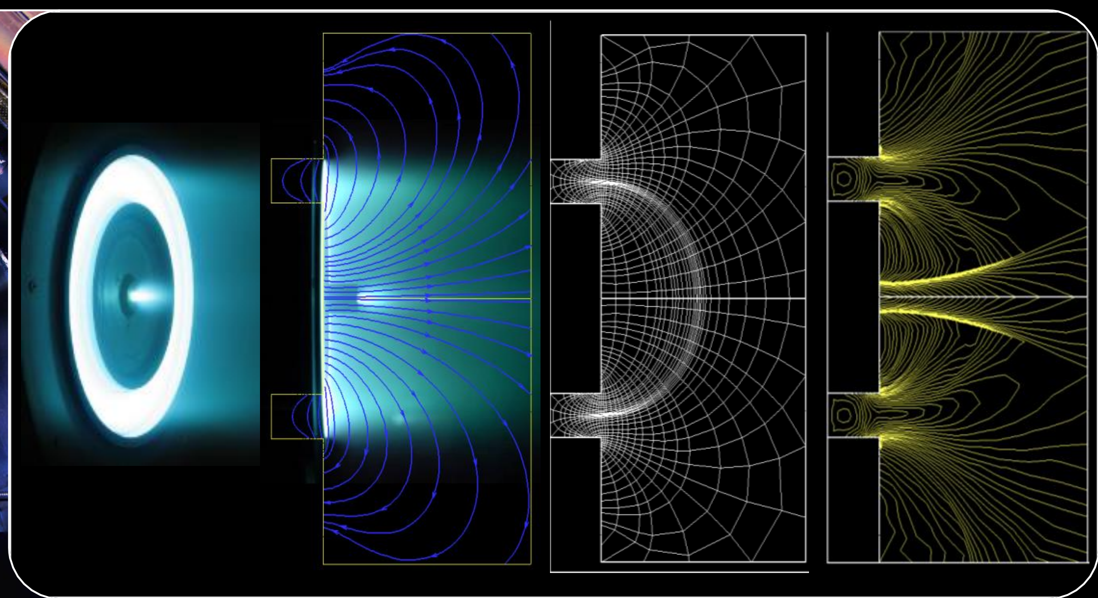
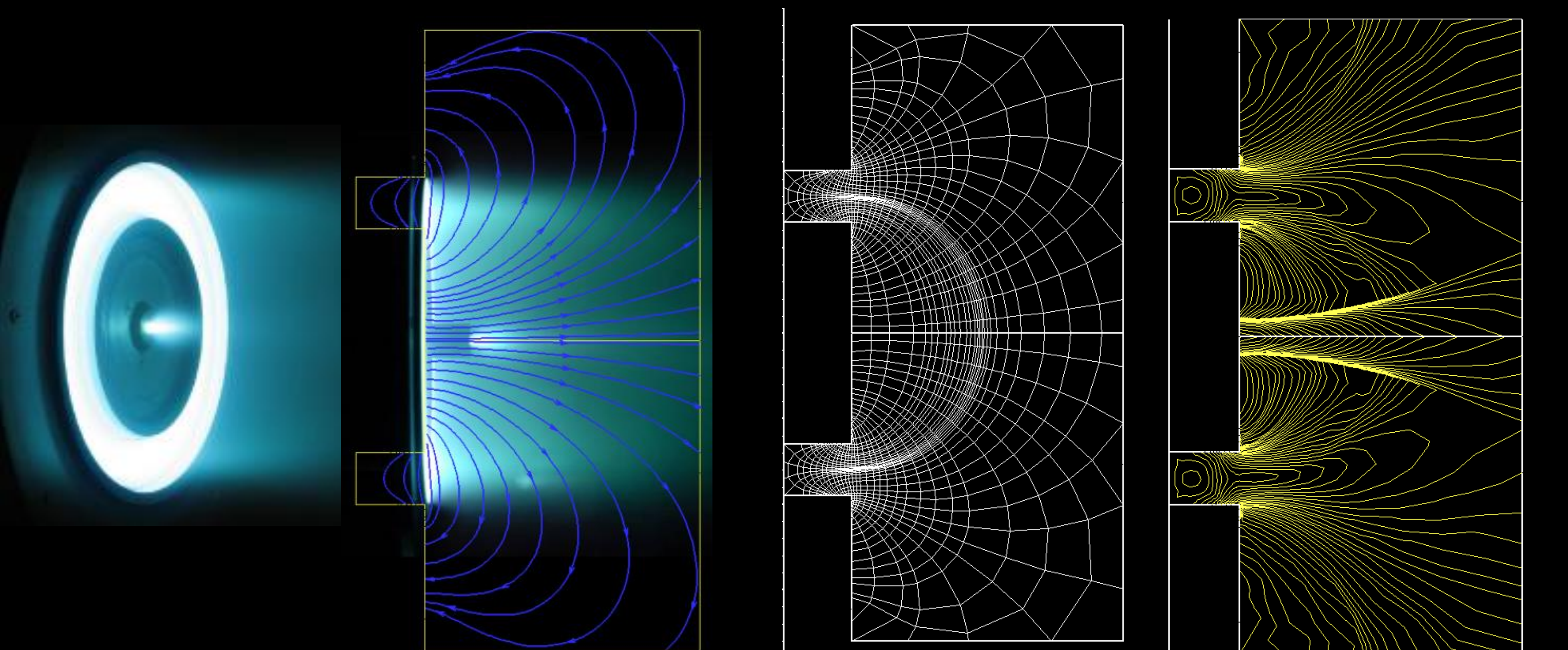
2018 ExB Workshop
Princeton, NJ, November 1-2, 2018





The 2-D axisymmetric (r-z) Code Hall2De

- Began development at JPL in 2008
- Discretization of all conservation laws on a magnetic field-aligned mesh
- Two components of the electron current density field accounted for in Ohm's law
- No statistical noise in the numerical solution of the heavy-species conservation laws
- Multiple ion populations allowed
- Large computational domain, extending several times the thruster channel length



6 kW Lab Hall thruster | Magnetic field streamlines | Hall2De computational mesh | Ion density line contours

Summary

- Can a first-principles model for the anomalous transport in electrons in Hall thrusters be successfully implemented in a fluid code?

- Can a fluid code predict low frequency dynamics (i.e., breathing mode oscillations)? How sensitive these oscillations are to models (i.e., wall losses, anomalous transport, thermal conductivity)?

Closure of Hall2De equations requires anomalous collision frequency

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$$\frac{\partial n_i}{\partial t} + \nabla\cdot(n\mathbf{u})_i = \dot{n}, \qquad \dot{n} = \int \left(\dot{f}_i\right)_c d\mathbf{v}\Big|_{\text{inelastic}}$$

$$\frac{\partial}{\partial t}(nm\mathbf{u})_i + \nabla\cdot(nm\mathbf{u}\mathbf{u})_i = q_i n_i \mathbf{E} - \nabla p_i + \mathbf{R}_i$$

$$\mathbf{R}_i \approx -\sum_{s\neq i} n_i m_i \nu_{is}(\mathbf{u}_i - \mathbf{u}_s) + \int m_i \mathbf{v}\left(\dot{f}_i\right)_c d\mathbf{v}\Big|_{\text{inelastic}}$$

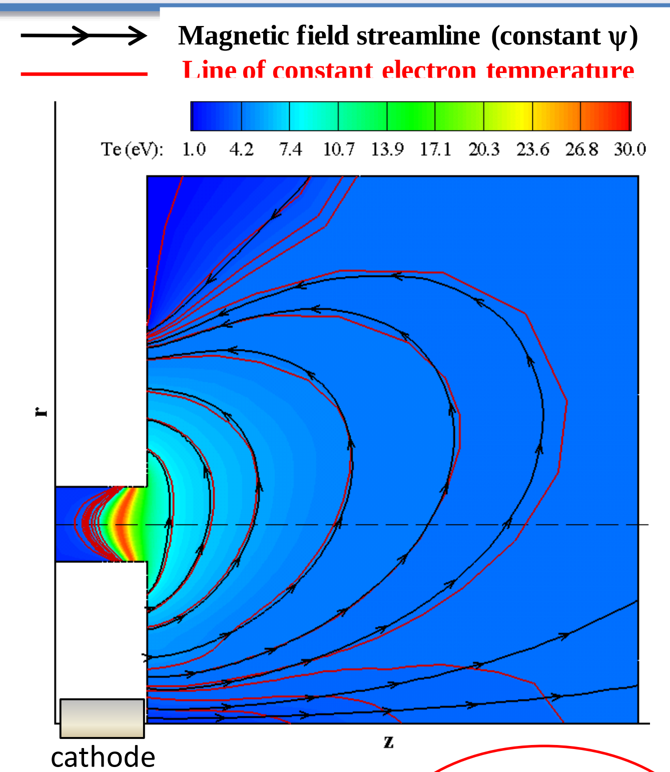
$$\nabla\cdot(\mathbf{j}_e + \mathbf{j}_i) = 0$$

$$\cancel{n_e m_e \frac{D\mathbf{u}_e}{Dt}} = -en_e(\mathbf{E} + \mathbf{u}_e\times\mathbf{B}) - \nabla p_e + \mathbf{R}_e$$

$$\rightarrow E_{//} = \eta j_{e//} - \frac{\nabla_{//} p_e}{en_e} + \eta_{ei}\bar{j}_{i//}, \quad E_\perp = \eta\left(1+\Omega_e^{\,2}\right) j_{e\perp} - \frac{\nabla_\perp p_e}{en_e} + \eta_{ei}\bar{j}_{i\perp}$$

$$\eta \equiv \frac{m_e\left(\nu_c + \overset{?}{\nu_a}\right)}{e^2 n_e}$$

$$\frac{3}{2} e n_e \frac{\partial T_e}{\partial t} = \mathbf{E}\cdot\mathbf{j}_e + \nabla\cdot\left(\frac{5}{2}T_e\mathbf{j}_e + \boldsymbol{\kappa}_e\cdot\nabla T_e\right) - \frac{3}{2}T_e\nabla\cdot\mathbf{j}_e - \sum_s \dot{n}e\left(\varepsilon + \frac{3}{2}T_e\right) + Q_e^T$$



Major assumptions of first-principles model

- Model tracks "wave action" as representative quantity of wave magnitude
- Maximum wave action established by saturation value
- Relationship between wave action and anomalous collision frequency is non-linear in region where electrons are not Maxwellian (Fig. 1)
- **Floor** value of anomalous collision frequency assumed in non-Maxwellian regions: minimum anomalous collision frequency for which electron drift velocity < electron thermal speed (Fig. 2)

$$K_e = \frac{e n_0 L}{|j_\perp| \tau_{ee}}$$

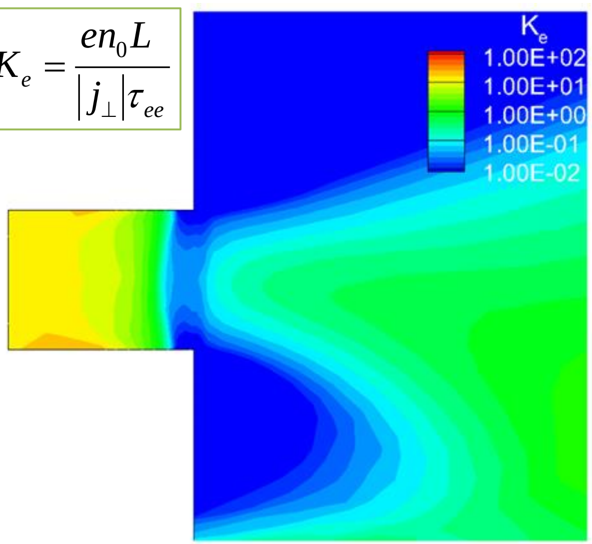


Fig. 1: For K_e<< 1, electrons are not Maxwellian

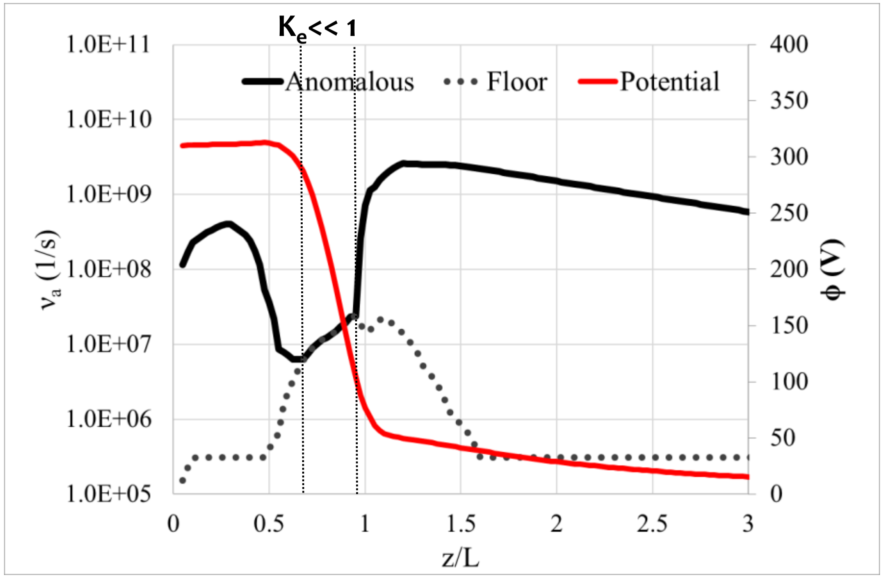
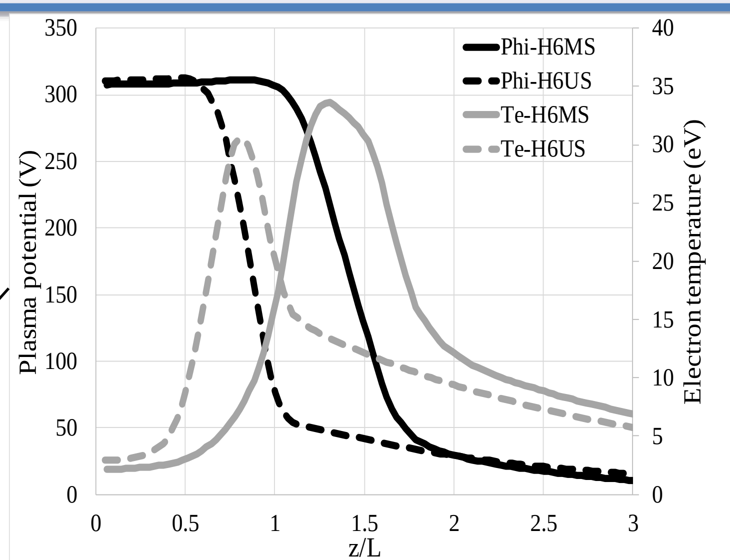
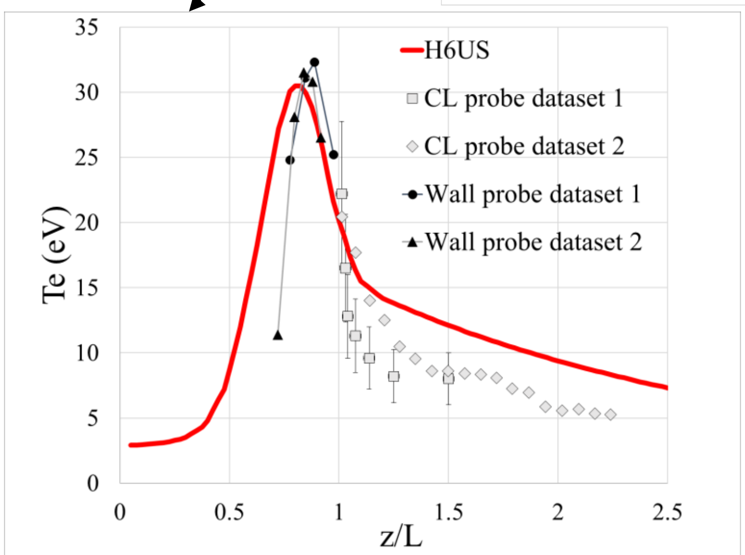


Fig. 2: Anomalous collision frequency predicted by first principles model

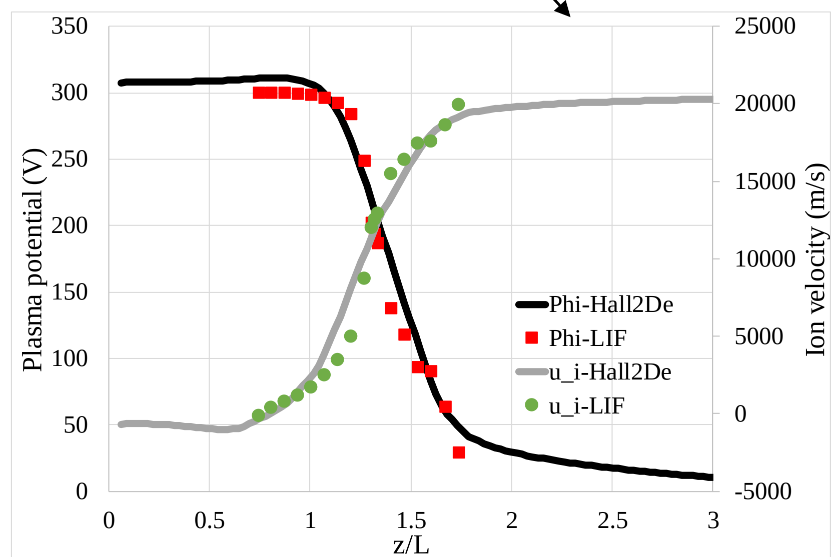
Can the first-principles model be applied successfully to multiple thrusters?



H6US comparison with experiments



H6MS comparison with experiments



Lopez Ortega et al., "Application of a first-principles anomalous transport model for electrons to multiple Hall thrusters and operating conditions", JPC 2018

Lopez Ortega et al., "A first-principles model based on saturation of the electron cyclotron drift instability for electron transport in hydrodynamics simulations of Hall thruster plasmas", IEPC 2017

Can a fluid code capture low frequency oscillations?

- We use a 1-D version of Hall2De to gain insight on low frequency oscillations
- Linearized system of equations using mass conservation of ions and neutrals produces criterion for positive growth rate

Ion velocity

Neutral velocity

Wall loss frequency for ions

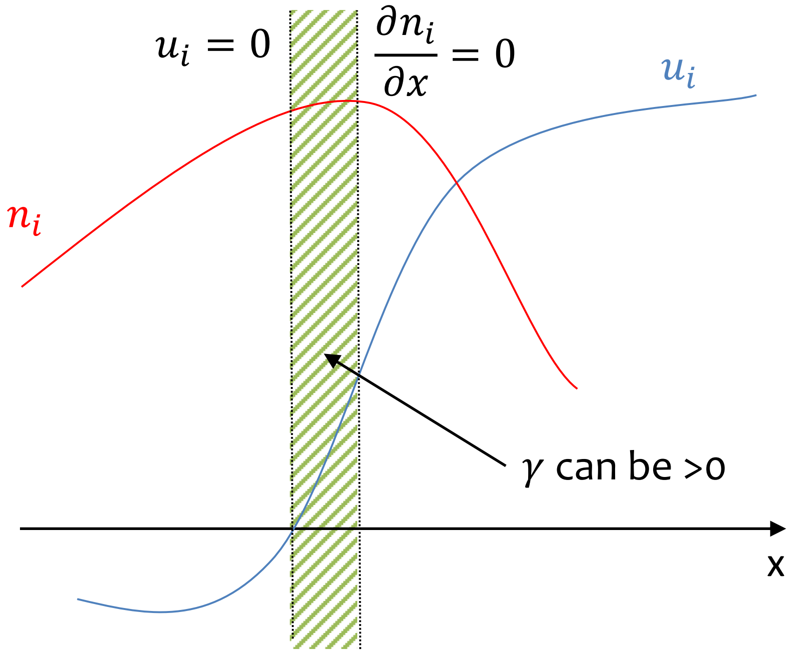
Growth rate

$$\gamma = \frac{1}{2}\left(\boxed{\frac{u_i}{n_i}\frac{\partial n_i}{\partial x}} + \underbrace{\frac{u_n}{n_n}\frac{\partial n_n}{\partial x} - \frac{\Gamma_i n_i}{n_n}}_{\text{Always} < 0}\right)$$

Neutral density

Ion density

Must be positive for $\gamma > 0$



Assuming oscillations are small, the 0-th order steady-state solution for ion conservation can be written as:

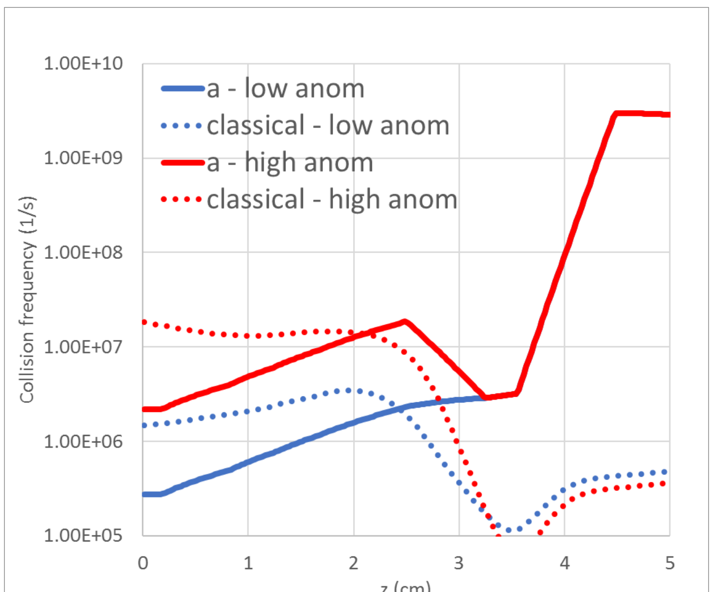
$$\frac{u_i}{n_i}\frac{\partial n_i}{\partial x} = \underbrace{-\frac{\partial u_i}{\partial x} - \Gamma_i}_{} + \underbrace{c_e\sigma(T_e)n_n}_{}$$

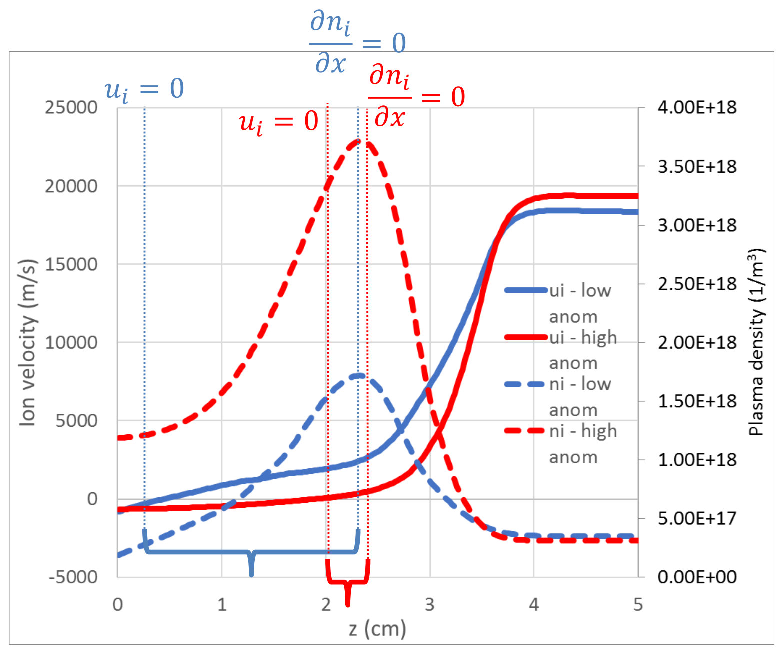
convection < ionization

Can a fluid code capture low frequency oscillations? What drives them?

- **Presence of oscillations is very sensitive to small changes in models that produce reasonable plasma solutions:**
  - Wall losses
  - Thermal conductivity
  - Collision cross sections
  - Ionization cross sections
  - Anomalous collision frequency

Example: anomalous collision frequency





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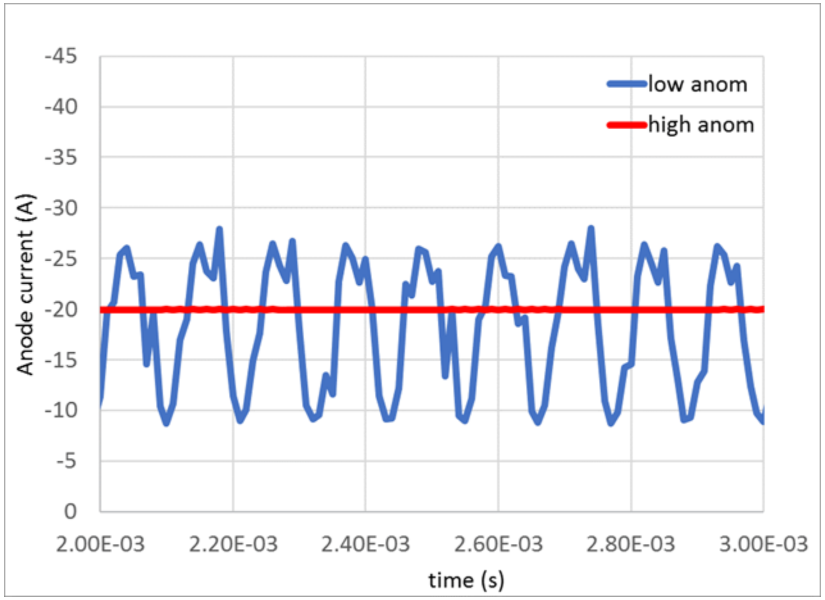

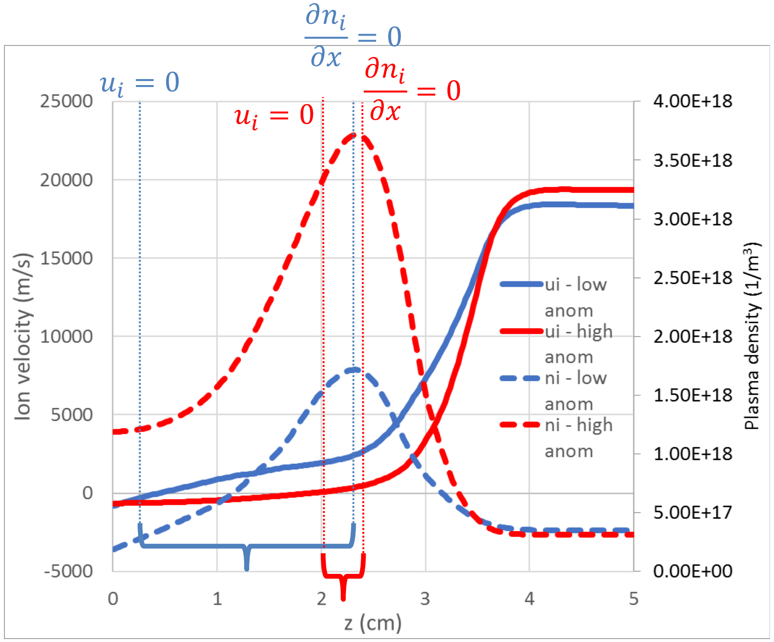

BACK UP

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Major assumptions of first-principles model

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$$\frac{\partial N_k}{\partial t} + \mathbf{u}_i \cdot \nabla N_k = 2\omega_{i,k,linear}\left(1 - \frac{N_k}{N_{k,sat}}\right)$$

Includes electron drift + Landau damping

$$N_{k,sat} = \frac{n_0 T_e}{4nc_s(1 + k^2\lambda_{De}^2)}$$

Limits plasma potential oscillations not to exceed T_e

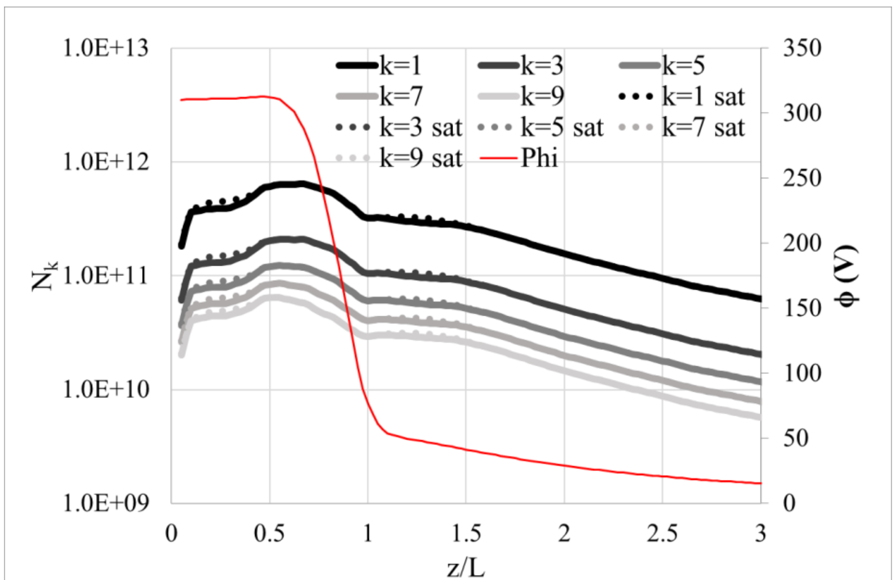
$$\nu_a = \frac{2e}{Nm_e n_0|\mathbf{u}_e - \mathbf{u}_i|}\sum_k kN_k\omega_{ie,k,nonlinear}$$

Number of discrete wavenumbers

Includes electron drift

$K_e \gg 1$ $\omega_{ie,k,nonlinear}$=$\omega_{ie,k,linear}$

$K_e \ll 1$ $\omega_{ie,k,nonlinear}$??



$$K_e = \frac{en_0 L}{|j_\perp|\tau_{ee}}$$

$$\frac{\partial N_k}{\partial t} + u_i \cdot \nabla N_k = 2\omega_{i,k,linear}\left(1 - \frac{N_k}{N_{k,sat}}\right)$$

Can a fluid code capture low frequency oscillations?

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Linearized system with analytical solution: ion density + neutral density equations in 1-D

$$\frac{Dn_i}{Dt} = -n_i\frac{\partial u_i}{\partial x} + \underbrace{kn_in_n}_{\text{ionization}} - \underbrace{\Gamma_i n_i}_{\text{wall losses}}$$

$$\frac{Dn_n}{Dt} = -kn_in_n + \Gamma_i n_i$$

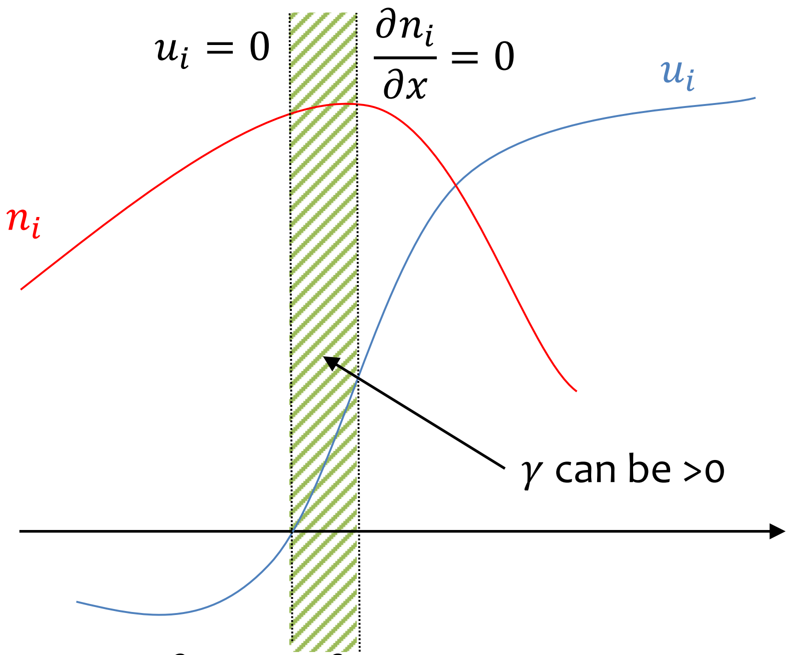
steady state

$$u_i\frac{\partial n_i}{\partial x} = -n_i\frac{\partial u_i}{\partial x} + kn_in_n - \Gamma_i n_i$$

$$u_n\frac{\partial n_n}{\partial x} = -kn_in_n + \Gamma_i n_i$$

linearized system

$$\frac{D}{Dt}\begin{pmatrix} n'_i \\ n'_n \end{pmatrix} = \begin{bmatrix} -\frac{\partial u_i}{\partial x} + kn_n - \Gamma_i & kn_i \\ -kn_n + \Gamma_i & -kn_i \end{bmatrix}\begin{pmatrix} n'_i \\ n'_n \end{pmatrix} = 0$$



Growth rate

$$\gamma = \frac{1}{2}\left(\boxed{\frac{u_i}{n_i}\frac{\partial n_i}{\partial x}} + \underbrace{\frac{u_n}{n_n}\frac{\partial n_n}{\partial x} - \frac{\Gamma_i n_i}{n_n}}_{<0}\right)$$

Frequency

$$\omega^2 = kn_i\frac{\partial u_i}{\partial x} - \gamma^2$$

$$\frac{u_i}{n_i}\frac{\partial n_i}{\partial x} = \underbrace{-\frac{\partial u_i}{\partial x} - \Gamma_i}_{\text{convection}} + \underbrace{kn_n}_{\text{ionization}}$$

convection < ionization